



磁化夸克-胶子在等离子体中的非均匀辐射

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Anisotropic Emission from Magnetized Quark-gluon Plasma

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Abstract: In this work, the polarization effects of a strongly magnetized quark-gluon plasma are studied at finite temperature. It is found that a background magnetic field can have a strong effect on the photon and dilepton emission rates. It affects not only the total rate but also the angular dependence. In particular, the Landau-level quantization leads to a nontrivial momentum dependence of the photon/dilepton anisotropic flow coefficient on transverse momentum. In the case of photon emission, nonzero coefficients v_n (with even n) have opposite signs at small and large values of the transverse momentum. Additionally, the v_n signs alternate with increasing n , and their approximate values decrease as $1/n^2$ in magnitude. The anisotropy of dilepton emission is well-pronounced only at large transverse momenta and small invariant masses. The corresponding v_n coefficients are of the same magnitude and show a similar sign-alternative pattern with increasing n as in the photon emission. It is proposed that the anisotropy of the photon and dilepton emission may serve as indirect measurements of the magnetic field.

Key words: heavy ion collision; electromagnetic probe; strong magnetic field; finite temperature theory

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0 Introduction

There has been a growing interest in the properties of Quantum chromodynamics (QCD) matter for the last half a century. The heavy-ion collision experiments at the Relativistic Heavy Ion Collider (RHIC) in Brookhaven and the Large Hadron Collider (LHC) at CERN give a strong experiment evidences to study such matter on the earth, especially the quark-gluon plasma (QGP) produced in heavy-ion collision experiments. The study of the fundamental properties of the corresponding deconfined state of matter is the broad goal of the ongoing program. Besides, in the noncentral heavy-ion collisions, the QGP are produced with a super strong magnetic field. In the early stages of the collision, the strength of the magnetic field could be estimated^[1-4]. However, how strong the magnetic field in the latter stages of the collision when the hot plasma forms and expands is still a puzzle. If it remains relatively strong, the magnetic field can trigger anomalous phenomena, modify the flow of plasma, cause new types of collective modes, and affect the emission of particles. In addition, the large background effects make huge difficulty on the observations of magnetic field from the experiments.

The electromagnetic probes (*i.e.*, photons and leptons) play a unique role in heavy ion collision experiments. Unlike the strongly interacting hadrons, they have long mean-free paths that greatly exceed the size of the fireballs created by the collisions. Thus, they carry invaluable information about the plasma directly to the detectors. Recently, several measurements of electromagnetic emissions were reported by both RHIC and LHC^[5-16]. In general, the thermal electromagnetic emission rates have been seen as thermometers of the QGP. In this study, we will introduce the thermal electromagnetic emission rates that can be perfect magnetometers for the hot QGP with a strong background magnetic field.

One has to be argued that, in a vanishing magnetic field case, the leading order photon production is given by the gluon-mediated $2 \rightarrow 2$ processes which are linear in the strong coupling constant α_s ^[17-23]. In strongly magnetized plasma, the photon rate is dominated by the following three single-photon processes: (i) the quark splitting ($q \rightarrow q + \gamma$), (ii) the antiquark splitting ($\bar{q} \rightarrow \bar{q} + \gamma$), (iii) the quark-antiquark annihilation ($q + \bar{q} \rightarrow \gamma$) which are no longer forbidden by the energy-momentum conservation. The photon rate is nonzero already at the leading zeroth order in α_s ^[24-26].

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For the dilepton, in a vanishing magnetic field, the thermal radiation from the QGP, the Drell–Yan process, and semileptonic decays of heavy quarks provide the dominant contributions to the dilepton rate in the intermediate range of the dilepton invariant masses. However, the signature effects of the magnetic field are the rate enhancement and strong anisotropy, whose measurements could provide valuable bounds on the field strength in the plasma produced by heavy-ion collisions^[27].

The main goal of this paper is to introduce the recent theoretical understanding of photon/dilepton emission from a strongly magnetized hot QGP. The corresponding results will reveal a nontrivial anisotropic photon/dilepton emission caused by the background magnetic field for a range of model parameters. As we will argue, such anisotropy carries information about the magnitude of the magnetic field at the early stages of heavy-ion collisions.

1 Anisotropic flow

In noncentral heavy-ion collisions, the anisotropic flow coefficients (v_n) are defined by the following Fourier decomposition of the azimuthal particle distributions^[28–29]:

$$E \frac{d^3N}{d^3p} = \frac{1}{2\pi} \frac{d^2N}{p_T dp_T dy} \left(1 + 2 \sum_{n=1}^{\infty} v_n \cos[n(\phi - \Psi_{RP})] \right), \quad (1)$$

where E is the particle energy, \mathbf{p} is the momentum, p_T is the transverse momentum, ϕ is the azimuthal angle, y is the rapidity, and Ψ_{RP} is the reaction plane angle. By definition:

$$v_n(p_T, y) = \langle \cos[n(\phi - \Psi_{RP})] \rangle, \quad (2)$$

where the angular brackets denote the average over all particles (or all events, or both) in a given bin of the transverse momentum (p_T) and rapidity (y). Note that the second coefficients in the Fourier decomposition v_2 , characterize the elliptic flow.

The direction of the magnetic field in a noncentral collision is (approximately) perpendicular to the reaction plane. In the paper, we assume z axis is the direction of the magnetic field, x - y is the reaction plane and the x axis points along the beam direction. The azimuthal angle ϕ measures the angle between the particle momentum \mathbf{p} and the reaction plane. The particle four-momentum is given by $p^\mu = (p^0, \mathbf{p})$. The transverse components of the particle momentum are given by

$$p_y = p_T \cos(\phi), \quad p_z = p_T \sin(\phi), \quad (3)$$

where $p_T = \sqrt{p_y^2 + p_z^2}$ is the magnitude of the transverse momentum.

For photon/dilepton, we assume that the mean free path is larger than the system size so that the particles leave the plasma region without reabsorption. The anisotropy

coefficients v_n can be evaluated from the differential distribution of particles as follows:

$$v_n = \frac{1}{\mathcal{R}_0} \int_0^{2\pi} \frac{d^3R}{p_T dp_T d\phi dy} \cos(n\phi) d\phi, \quad (4)$$

where the normalization factor is given by the particle production rate integrated over the angular coordinate ϕ , *i.e.*,

$$\mathcal{R}_0 = \frac{d^2R}{p_T dp_T dy} = \int_0^{2\pi} \frac{d^3R}{p_T dp_T dy d\phi} d\phi. \quad (5)$$

2 Thermal photons in a magnetized QGP

By making use of quantum field theory, the thermal photon production rate can be expressed in terms of the imaginary part of the retarded polarization tensor as follows^[30]:

$$p^0 \frac{d^3R}{dp_x dp_y dp_z} = -\frac{1}{(2\pi)^3} \frac{\text{Im}[\Pi_\mu^\mu(p)]}{\exp\left(\frac{p_0}{T}\right) - 1}. \quad (6)$$

Because of a quantizing background magnetic field, the quark and antiquark states in the one-loop photon polarization are characterized by the Landau-level quantum numbers, the integer indices n and n' . For simplicity, we will assume that the masses of both light quarks are the same, namely, $m_u = m_d = m = 5 \text{ MeV}$. The flavor-dependent quark charges are q_f , where $q_u = 2/3e$, $q_d = -1/3e$, and e is the absolute value of the electron charge. We define the modified fine structure constant $\alpha_f = q_f^2/(4\pi)$ for different flavors of quarks. $N_c = 3$ is the number of colors, T is the temperature of the system and n_F is the Fermi-Dirac distribution function.

The imaginary part of the (Lorentz-contracted) polarization tensor in the leading order with a background magnetic field can be written as follows^[26]:

$$\begin{aligned} \text{Im}[\Pi_{R,\mu}^\mu] = & \sum_{f=u,d} \frac{N_c \alpha_f}{2\pi l_f^4} \sum_{n>n'} g(n, n') \\ & \frac{\left[\theta(p_{-,f}^2 - p_0^2 + p_3^2) - \theta(p_0^2 - p_3^2 - p_{+,f}^2) \right]}{\sqrt{(p_{-,f}^2 + p_z^2 - p_0^2)(p_{+,f}^2 + p_z^2 - p_0^2)}} \mathcal{F}_{n,n'}^f - \\ & \sum_{f=u,d} \frac{N_c \alpha_f}{4\pi l_f^4} \sum_{n=0}^{\infty} \frac{g_0(n) \theta(p_0^2 - p_z^2 - p_{+,f}^2)}{\sqrt{(p_0^2 - p_z^2)(p_0^2 - p_z^2 - p_{+,f}^2)}} \mathcal{F}_{n,n}^f, \end{aligned} \quad (7)$$

where $\theta(x)$ is the Heaviside step function, the transverse momentum thresholds are given by

$$p_{\pm,f} = \left| \sqrt{m^2 + 2n|q_f B|} \pm \sqrt{m^2 + 2n'|q_f B|} \right|. \quad (8)$$

We also used the following shorthand notations:

$$g(n, n') = 2 - \sum_{s_1, s_2 = \pm} n_f \left[\frac{p_0}{2} + s_1 \frac{p_0(n - n')|q_f B|}{p_0^2 - p_z^2} + \right. \\ \left. s_2 \frac{|p_z|}{2(p_0^2 - p_z^2)} \sqrt{(p_0^2 - p_z^2 - p_{-,f}^2)(p_0^2 - p_z^2 - p_{+,f}^2)} \right], \\ g_0(n) = g(n, n), \quad (9)$$

and the explicit form of the corresponding function reads

$$\mathcal{F}_{n,n'}^f(\xi) = 8\pi(n + n' + m^2 \ell_f^2) \left[\mathcal{I}_{0,f}^{n,n'}(\xi) + \mathcal{I}_{0,f}^{n-1,n'-1}(\xi) \right] + \\ 8\pi \left[\frac{(p_0^2 - p_z^2) \ell_f^2}{2} - (n + n') \right] \left[\mathcal{I}_{0,f}^{n,n'-1}(\xi) + \mathcal{I}_{0,f}^{n-1,n'}(\xi) \right], \quad (10)$$

where functions $\mathcal{I}_{0,f}^{n,n'}(\xi)$ are defined in terms of the generalized Laguerre polynomials^[31] as follows:

$$\mathcal{I}_{0,f}^{n,n'}(\xi) = (-1)^{n+n'} e^{-\xi} L_n^{n'-n}(\xi) L_{n'}^{n-n'}(\xi), \quad (11)$$

$\xi = (k_\perp \ell_f)^2/2$ and the flavor-specific magnetic length $\ell_f = 1/\sqrt{|q_f B|}$.

The total rate integrated over the angular coordinate are plotted in Fig. 1 for both two different magnetic fields and two different temperatures: 1) $|eB| = m_\pi^2$, $T = 200$ MeV (red dashed line), 2) $|eB| = 5m_\pi^2$, $T = 350$ MeV (purple dotted line), 3) $|eB| = m_\pi^2$, $T = 350$ MeV (black solid line), 4) $|eB| = 5m_\pi^2$, $T = 350$ MeV (Blue dot-dashed line). In the case of the stronger field, $|eB| = 5m_\pi^2$, there are indeed well-resolved peaks in the photon production rate at $p_T \simeq 0.06$ GeV when $T = 0.2$ GeV and at $p_T \simeq 0.04$ GeV when $T = 0.35$ GeV. For each case, the transverse momenta were taken in the range between 0.01 GeV to 1 GeV with discretization step 0.01 GeV, and the azimuthal angle was considering the range between $10^{-4} \frac{\pi}{2}$ and $\frac{\pi}{2} - 10^{-4} \frac{\pi}{2}$ with the discretization step $10^{-3} \frac{\pi}{2}$.

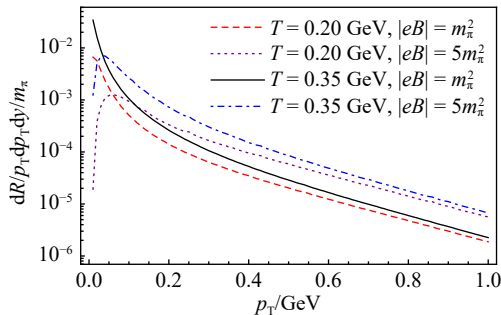


Fig. 1 The integrated photon production rate as a function of the transverse momentum p_T for two different magnetic field: $|eB| = m_\pi^2$, $|eB| = 5m_\pi^2$ and two different temperatures: $T = 200$ MeV, $T = 350$ MeV^[24, 26]. (color online)

As argued in Ref. [26], there also should have similar peaks appear in the case of weaker fields. The existence of such maxima is a necessary consequence of the Landau-level quantization of quark states in a strongly magnetized plasma. However, because of the numerical calculation dif-

ficulties at tiny p_T , the peak is not showing here. The maxima are larger when the temperature is higher and the magnetic field is weaker. In addition, when the transverse momentum increases further, the photon production rate starts to decrease quickly. The higher temperature and larger magnetic field induced a higher rate at larger p_T .

To better understand the role of the magnetic field on the direct photon emission from hot quark-gluon plasma, we show the plot of the ellipticity v_2 dependence on the transverse momentum in Fig. 2. For all the physics parameters we are plotting here, we found in all the cases we have the following feature. At large p_T , the ellipticity reaches and saturates at a relatively large positive value, i.e., $v_{2,\max} \simeq 0.2$. At small p_T regime $p_T \lesssim \sqrt{|eB|}$, the ellipticity become negative. The sign of v_2 changes at around $p_T \simeq \sqrt{|eB|}$. The corresponding results mean, the emission rate at small values of p_T has an overall tendency to peak at $\phi = \frac{\pi}{2}$, i.e., in the direction perpendicular to the reaction plane. When the value of p_T is larger than about $\sqrt{|eB|}$, the emission tends to be highest at $\phi = 0$, i.e., in the direction along the reaction plane. Such anisotropic emission is caused by a strong magnetic field and has nothing to do with the hydrodynamic behavior of the quark-gluon plasma (see Ref. [24] for details).

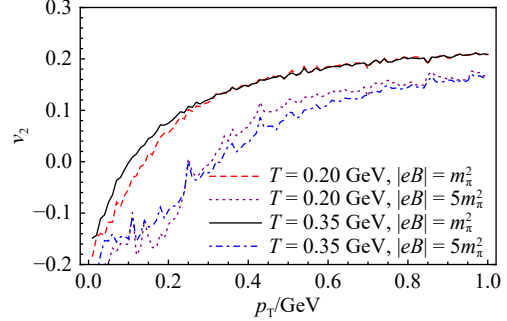


Fig. 2 Ellipticity of the photon production as a function of the transverse momentum p_T for two different magnetic fields: $|eB| = m_\pi^2$, $|eB| = 5m_\pi^2$ and two different temperatures: $T = 200$ MeV, $T = 350$ MeV^[24, 26]. (color online)

3 Dileptons in a magnetized QGP

The dilepton rate is given by^[32]

$$dR_{ll} = 2\pi e^2 e^{-\beta k_0} L_{\mu\nu}(Q_1, Q_2) \rho^{\mu\nu}(k_0, \mathbf{k}) \frac{d^3 q_1}{(2\pi)^3 E_1} \frac{d^3 q_2}{(2\pi)^3 E_2}, \quad (12)$$

where the three-momenta and energies of the two leptons are denoted by \mathbf{q}_i and E_i with $i=1,2$, respectively. One should notice here, to distinguish with photon in previous section, we use $k^\mu = (k_0, \mathbf{k})$ as the four-momenta of the dilepton in this section. To the leading linear order in the electromagnetic coupling constant, the electromagnetic spectral function $\rho^{\mu\nu}(k_0, \mathbf{k})$ is expressed in terms of the imaginary part of the photon polarization tensor as follows:

$$\rho^{\mu\nu}(k_0, \mathbf{k}) = -\frac{1}{\pi} \frac{e^{\beta k_0}}{e^{\beta k_0} - 1} \frac{\text{Im}[\Pi^{\mu\nu}(k_0, \mathbf{k})]}{K^4}. \quad (13)$$

In Eq. (12), the leptonic tensor for the final plane-wave states has the following explicit form:

$$L_{\mu\nu}(Q_1, Q_2) = \frac{1}{4} \sum_{\text{spins}} \text{tr}[\bar{u}(Q_2) \gamma_\mu v(Q_1) \bar{v}(Q_1) \gamma_\nu u(Q_2)]. \quad (14)$$

By using the explicit form of the leptonic tensor and setting the lepton masses m_l to zero, the corresponding differential rate reads^[32]

$$\frac{dR_{ll}}{d^4K} = \frac{\alpha}{12\pi^4} \frac{n_B(p_0)}{M^2} \text{Im}[\Pi_\mu^\mu(k_0, \mathbf{k})], \quad (15)$$

where $\alpha \equiv e^2/(4\pi) = 1/137$ is the fine structure constant and $n_B(k_0) = (e^{k_0/T} - 1)^{-1}$ is the Bose-Einstein distribution function. $M^2 = K^2 = k^2 \equiv k_0^2 - k_\perp^2 - k_z^2$ is the square of the invariant mass of the lepton pair. By definition, $k_\perp = \sqrt{k_x^2 + k_y^2}$ is the magnitude of the momentum component perpendicular to the magnetic field. Note that $d^4K = M dM k_T dk_T dy d\phi$, where $p_T = \sqrt{k_y^2 + k_z^2}$ is the transverse momentum (with respect to the beam direction) and $y = \frac{1}{2} \ln \frac{k_0 + k_z}{k_0 - k_z}$ is the rapidity.

After applying the imaginary part of the Lorentz-contracted photon polarization tensor in Eq. (7), the final result reads

$$\begin{aligned} \frac{dR_{ll}}{d^4K} &= \frac{\alpha^2 N_c}{48\pi^5} \frac{n_B(\Omega)}{M^2} \sum_{f=u,d} \frac{q_f^2}{f^4} \\ &\left[\sum_{n=0}^{\infty} \frac{g_0(n) \theta(\sqrt{M^2 + k_\perp^2} - k_{+,f})}{\sqrt{(M^2 + k_\perp^2) [M^2 + k_\perp^2 - k_{+,f}^2]}} \mathcal{F}_{n,n}^f(\xi) - \right. \\ &2 \sum_{n>n'}^{\infty} \frac{g(n, n') \theta(k_{-,f} - \sqrt{M^2 + k_\perp^2}) \mathcal{F}_{n,n'}^f(\xi)}{\sqrt{[k_{-,f}^2 - (M^2 + k_\perp^2)] [k_{+,f}^2 - (M^2 + k_\perp^2)]}} + \\ &\left. 2 \sum_{n>n'}^{\infty} \frac{g(n, n') \theta(\sqrt{M^2 + k_\perp^2} - k_{+,f}) \mathcal{F}_{n,n'}^f(\xi)}{\sqrt{[k_{-,f}^2 - (M^2 + k_\perp^2)] [k_{+,f}^2 - (M^2 + k_\perp^2)]}} \right]. \quad (16) \end{aligned}$$

In addition, it is useful to compare the results with the corresponding rate in the zero-field limit. In the Born approximation, the rate is given by^[33]

$$\frac{dR_{ll, \text{Born}}}{d^4k} = \frac{5\alpha^2 T}{18\pi^4 |\mathbf{k}|} n_B(k_0) \ln \left(\frac{\cosh \frac{k_0 + |\mathbf{k}|}{4T}}{\cosh \frac{k_0 - |\mathbf{k}|}{4T}} \right), \quad (17)$$

where the massless quarks and leptons are assumed.

The dependence of the rate on the invariant mass is presented in Fig. 3. The corresponding data was calculated for the whole range of invariant masses between $M_{\min} = 0.02$ GeV and $M_{\max} = 1$ GeV, using the discretization step $\Delta M = 0.01$ GeV. Individual panels show the results for two representative choices of temperature, *i.e.*,

$T = 0.2$ GeV and $T = 0.35$ GeV, and two values of the magnetic field, *i.e.*, $|eB| = m_\pi^2$ and $|eB| = 5m_\pi^2$. Each panel contains the rates for the same set of fixed values of the transverse momenta, *i.e.*, $k_T = 0$ (black), $k_T = 0.1$ GeV (blue), $k_T = 0.2$ GeV (orange), $k_T = 0.5$ GeV (green), and $k_T = 1$ GeV (red). For comparison, we also show the zero-field Born rate (dashed lines).

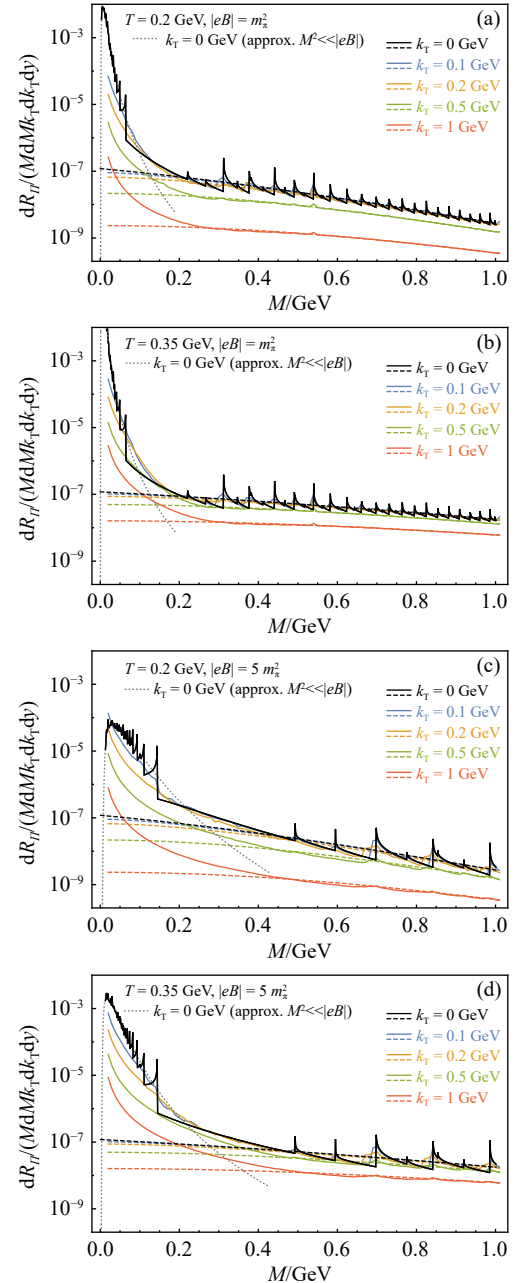


Fig. 3 The integrated dilepton rate as a function of the dilepton invariant mass M for several fixed values of the transverse momentum $k_T = 0, 0.1, 0.2, 0.5, 1$ GeV, two values of the temperature, *i.e.*, $T = 0.2$ GeV and $T = 0.35$ GeV, and two values of the magnetic field, *i.e.*, $|eB| = m_\pi^2$ and $|eB| = 5m_\pi^2$. For comparison, the dashed lines represent the zero-field Born rate and the gray dotted lines show the approximate rate at $k_T = 0$ and $M \ll \sqrt{|eB|}$ ^[27]. (color online)

By comparing the overall profiles of dilepton rates in a strongly magnetized plasma with the benchmark zero-field Born rate, the rates at $B \neq 0$ remain about the same on average as those at $B=0$ when the invariant mass is sufficiently large, *i.e.*, $M \gtrsim \sqrt{|eB|}$. On the other hand, the magnetic field has a dramatic effect on the dilepton production in the region of small invariant masses, *i.e.*, $M \lesssim \sqrt{|eB|}$, where the rates are strongly enhanced. As seen in Fig. 3, the rate can increase by several orders of magnitude when M decreases only by half. The dilepton rate is on-average a decreasing function of the invariant mass M . Generically, the rate in the magnetized QGP approaches the zero-field Born result (17) when the invariant mass is sufficiently large (*i.e.*, $M \gg \sqrt{|eB|}$). By comparing the plots in Fig. 3 for different values of the transverse momenta, we see that the dilepton rate tends to decrease with increasing k_T . As

one can verify, the suppression of the rate at large M or k_T (or both) comes primarily from the overall Bose distribution function $n_B(\Omega)$ in Eq. (16).

The dependence of the ellipticity parameter v_2 on the dilepton invariant mass M is shown in Fig. 4. In the case of small invariant masses, $M \lesssim \sqrt{|eB|}$, there is a clear tendency of v_2 to become positive. The latter is particularly well pronounced at large k_T , which is analogous to the photon emission. Such a behavior is because the angular dependence of the rate are systematically larger for the azimuthal directions near $\phi \approx 0$ (in the reaction plane) and systematically smaller for $\phi \approx \pi/2$ (out of the reaction plane). On the other hand, the value of the ellipticity parameter v_2 for the dilepton rate at small invariant masses, $M \lesssim \sqrt{|eB|}$, and large transverse momenta is positive.

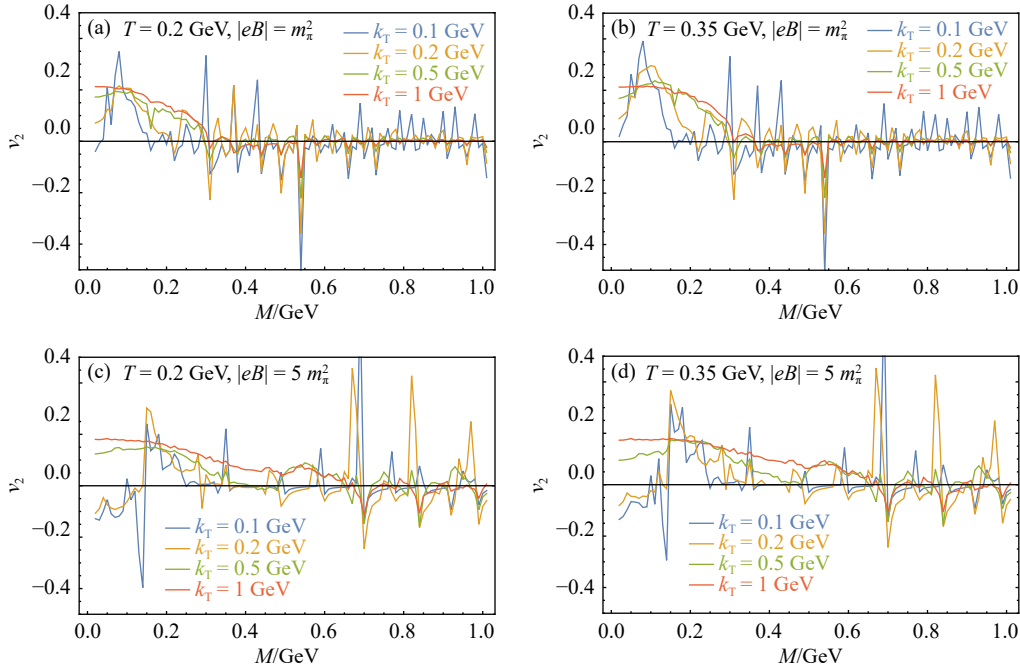


Fig. 4 The dilepton emission ellipticity as a function of the dilepton invariant mass M for two temperatures, *i.e.*, $T=0.2$ GeV and $T=0.35$ GeV, and two magnetic fields, *i.e.*, $|eB|=m_\pi^2$ and $|eB|=5m_\pi^2$. (color online)

Each panel shows the results for four fixed transverse momenta, *i.e.*, $k_T=0.1$ GeV (blue), $k_T=0.2$ GeV (orange), $k_T=0.5$ GeV (green), and $k_T=1$ GeV (red)^[27].

The overall ellipticity is harder to discern from the data at small values of the transverse momenta, *i.e.*, $k_T=0.1$ GeV (blue lines) and $k_T=0.2$ GeV (orange lines). For sufficiently small invariant masses, $M \lesssim \sqrt{|eB|}$, the ellipticity parameter appears to be generally nonvanishing. However, as seen from Fig. 4, its sign may change from being positive at intermediate values of M (*i.e.*, $M \approx \sqrt{|eB|}$), to being negative at sufficiently small values of M (*i.e.*, $M \ll \sqrt{|eB|}$). This is particularly clear in the case of the stronger magnetic field $|eB|=5m_\pi^2$. It should be also noted that the average ellipticity parameter v_2 is consistent with zero for sufficiently large invariant masses, $M \gtrsim \sqrt{|eB|}$.

4 Higher-order anisotropy coefficients

In this section, we will introduce the higher-order anisotropy coefficients v_4 and v_6 in the photon and dilepton emission from a hot magnetized quark-gluon plasma. By using the definition of v_n in Eq. (4), we get the numerical results of v_4 and v_6 .

The numerical results for v_4 and v_6 in the photon emission are shown in Figs. 5 and 6. The different panels display the results for two different magnetic fields, $|eB|=m_\pi^2$ and $|eB|=5m_\pi^2$, respectively. In both cases, the blue solid and the red dashed lines correspond to two fixed

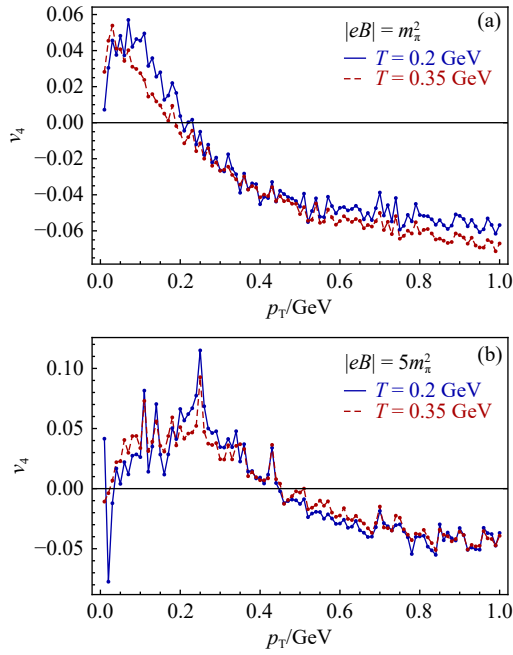


Fig. 5 Anisotropic coefficient ν_4 for the photon emission as a function of the transverse momentum p_T for two different temperatures, $T=0.2$ GeV (blue) and $T=0.35$ GeV (red), and two different strengths of the magnetic field, $|eB|=m_\pi^2$ and $|eB|=5m_\pi^2$ [34]. (color online)

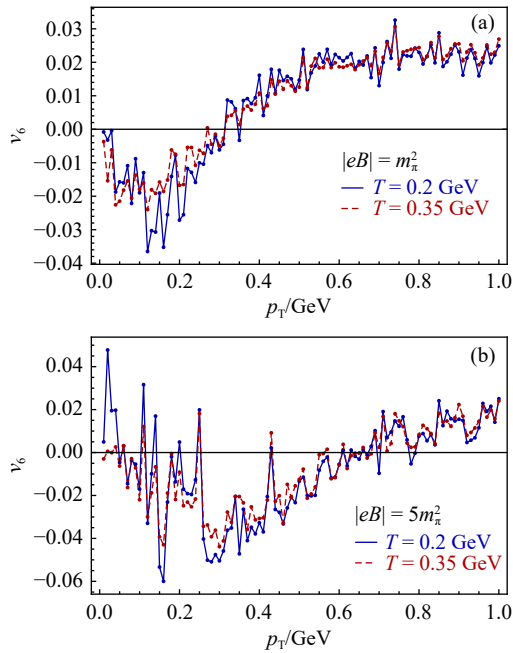


Fig. 6 Anisotropic coefficient ν_6 for the photon emission as a function of the transverse momentum p_T for two different temperatures, $T=0.2$ GeV (blue) and $T=0.35$ GeV (red), and two different strengths of the magnetic field, $|eB|=m_\pi^2$ and $|eB|=5m_\pi^2$ [34]. (color online)

temperatures, *i.e.*, $T=0.2$ GeV and $T=0.35$ GeV, respectively. At relatively small momenta, $p_T \lesssim \sqrt{|eB|}$, $\nu_4(\nu_6)$ tends to be positive(negative). However, it becomes negative(positive) for $p_T \gtrsim \sqrt{|eB|}$. The absolute values of $\nu_4(\nu_6)$ are of the order of 0.05(0.02). Together with the results of

ν_2 , we note the alternating signs of the anisotropy coefficients ν_n with increasing n (all coefficients with odd n vanish) and the overall scaling of the magnitude goes as $1/n^2$.

The numerical results for the dilepton $\nu_4(\nu_6)$ as a function of the invariant mass is shown in Fig. 7(8). The four panels present the results for two temperatures, *i.e.*, $T=0.2$ GeV and $T=0.35$ GeV, and two magnetic fields $|eB|=m_\pi^2$ and $|eB|=5m_\pi^2$. At small invariant masses, the coefficient $\nu_4(\nu_6)$ tends to be negative(positive) with abso-

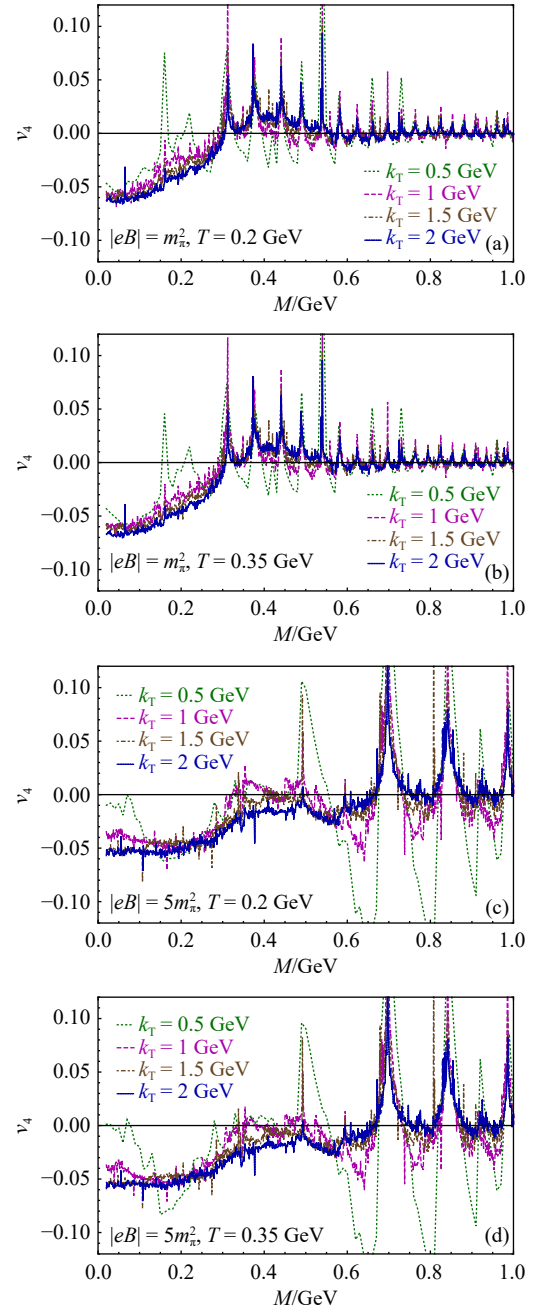


Fig. 7 Anisotropic coefficient ν_4 for the dilepton emission as a function of the invariant mass M for several fixed values of the transverse momentum k_T . The panels correspond to $|eB|=m_\pi^2$, $|eB|=5m_\pi^2$ for $T=0.2$ GeV and $T=0.35$ GeV [34]. (color online)

lute values of about 0.05(0.02). They are comparable to the $v_4(v_6)$ values in the photon emission at large transverse momenta, see Figs. 5 and 6. A nonvanishing v_4 and v_6 are barely resolved for the intermediate momentum $k_T=0.5$ GeV, especially in the case of the stronger field $|eB|=5m_\pi^2$. It is not surprising as the corresponding k_T is comparable to $\sqrt{|eB|}$. Nevertheless, the trend becomes un-

ambiguous for the larger values of k_T . As anticipated, both v_4 and v_6 have a weak temperature dependence at sufficiently large transverse momenta. In addition to the alternating sign pattern and the hierarchy of coefficients $v_n \propto 1/n^2$ in the region of small invariant masses, *e.g.*, we see well-pronounced modulations in the v_n dependence on the invariant mass.

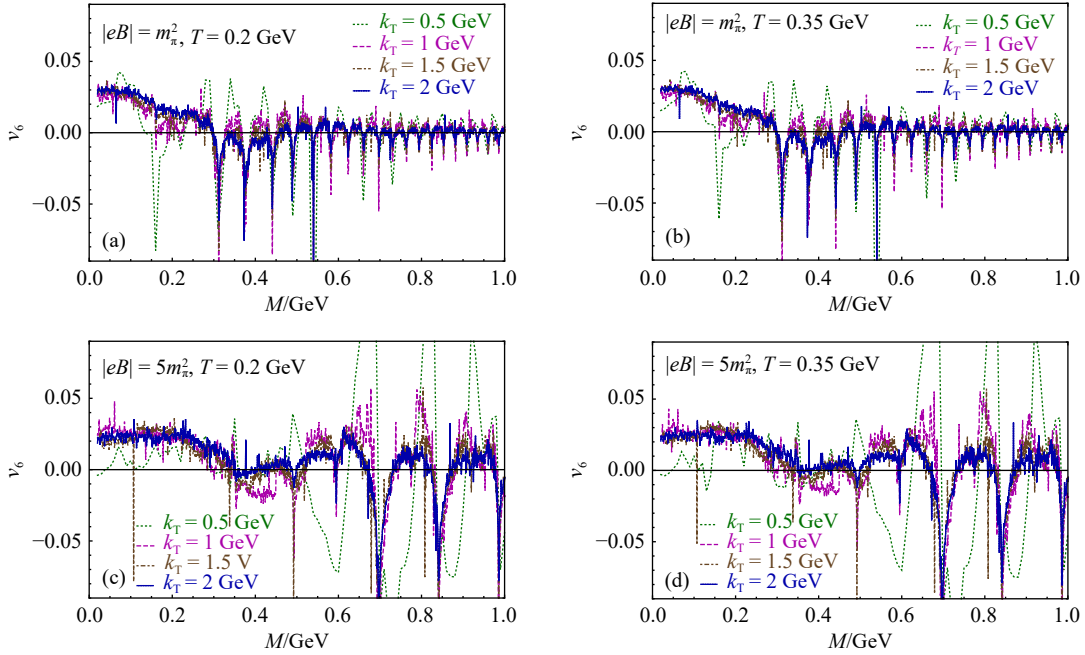


Fig. 8 Anisotropic coefficient v_6 for the dilepton emission as a function of the invariant mass M for several fixed values of the transverse momentum k_T . The panels correspond to $|eB|=m_\pi^2$, $|eB|=5m_\pi^2$ for $T=0.2$ GeV and $T=0.35$ GeV [34]. (color online)

5 Summary

The electromagnetic probes in relativistic heavy-ion collisions provide an ideal environment to study the properties of QCD matter under extreme conditions. By using the Landau-level representation for the imaginary part of the photon polarization tensor, we got an explicit expression for the photon/dilepton emission rate from a hot QGP in a quantizing background magnetic field.

For the photon emission from a strongly magnetized hot quark-gluon plasma, there is a nonzero ellipticity coefficient v_2 that depends on the transverse momentum. v_2 is negative at small momenta, $p_T \lesssim \sqrt{|eB|}$, and positive at large momenta, $p_T \gtrsim \sqrt{|eB|}$. While the ellipticity coefficient v_2 is an overall growing function of p_T , it reaches a relatively high positive value $v_{2,\max} \simeq 0.2$ at large p_T . One of the most exciting thing is the negative value of v_2 at small p_T , this result differs from the one in any hydrodynamic or transport calculations that predict positive v_2 for any p_T [35]. If such a negative value could be measured, this definitely an evidence of the existent magnetic field in the QGP plasma. Also, the higher-order anisotropy coefficients are as follows: $v_4 \simeq +0.05$ and $v_6 \simeq -0.02$. At large

momenta (*i.e.*, $p_T \gtrsim \sqrt{|eB|}$), the signs of v_n reverse, but the absolute values remain about the same, *i.e.*, $v_4 \simeq -0.05$ and $v_6 \simeq +0.02$. Combining these findings with the v_2 results, we see that the signs of even coefficients v_n alternate. The absolute values gradually decrease with increasing n in each kinematic region. Quantitatively, the scaling appears to go as $v_n \propto 1/n^2$.

The dilepton emission also has a noticeable anisotropy. It is well pronounced only in the kinematic regime with large transverse momenta (*i.e.*, $k_T \gtrsim \sqrt{|eB|}$) and small invariant masses (*i.e.*, $M \lesssim \sqrt{|eB|}$). The signs and absolute values of the anisotropy coefficients are as follows: $v_4 \simeq +0.2$, $v_4 \simeq -0.05$ and $v_6 \simeq +0.02$. We see that the signs of even coefficients v_n alternate, and their absolute values decrease with increasing n . The quantitative scaling is similar to that in the photon emissions. One should be noticed, under optimal conditions, the magnitude of v_2 could be as large as 0.2. If such large values are measured in an experiment, they will most likely indicate the presence of a nonzero magnetic field in the QGP plasma.

Besides, the background field strongly enhances the rate at small values of the dilepton invariant mass ($M \lesssim \sqrt{|eB|}$). At large values of the invariant mass

($M \gtrsim \sqrt{|eB|}$), of course, the role of the magnetic fields decreases, and the results gradually approach the isotropic zero-field Born rate. Such significant enhancement of the integrated dilepton rate at small invariant masses is a unique signature of a nonzero background magnetic field. Thus, measuring the corresponding rate at $M \lesssim 0.2$ GeV, for example, could provide sufficient information to confirm or rule out the fields of order $|eB| \simeq m_\pi^2$ in relativistic heavy-ion collisions.

It should be noticed, that although the motivation of this study was triggered by potential applications in heavy-ion physics, our main results may also find applications in astrophysics, where relativistic QED plasmas are common in such environments.

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磁化夸克-胶子在等离子体中的非均匀辐射

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摘要: 通过研究有限温度下强磁化夸克-胶子等离子体的极化效应, 发现背景磁场对光子和双轻子发射率有很强的影响。它不仅影响总发射率, 还影响角度相关性。特别是, 朗道能级量子化导致光子/双轻子各向异性流系数对横向动量的非平凡动量依赖性。在光子发射的情况下, 非零系数 v_n (n 为偶数) 在横向动量的很小和很大处具有相反的符号。此外, v_n 的正负随着 n 的增加交替出现, 其近似绝对值正比于 $1/n^2$ 。双轻子发射的各向异性仅在大横向动量和小不变质量时才会明显出现。相应的 v_n 系数具有相同的量级, 并且随着 n 的增加而显示出与光子发射类似的正负交替模式。光子和双轻子产生过程的各向异性也可以作为对磁场的测量一种重要的间接手段。

关键词: 重离子碰撞; 电磁探针; 强磁场; 有限温度场论

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